volume would increase in value as a reference book if the discussion of the literature references were more systematic and if the index were more exhaustive. The book is accessible to the nonspecialist, but requires close reading at certain points.

R. A. Zuidwijk
E-mail: zuidwijk@cwi.nl

S. J. Gardiner, *Harmonic Approximation*, London Mathematical Society Lecture Notes Series **221**, Cambridge Univ. Press, Cambridge UK, 1995, xiii + 132 pp.

Weierstrass' theorem (1885) that every continuous function on a compact interval can be approximated uniformly by polynomials is now just over a century old. This result can be considered as one of the foundations of polynomial approximation. Possible generalizations are approximation with more general analytic functions and approximation on more general compact sets of the complex plane. Curiously, Runge published in the same year his approximation theorem in a more general setting: if Ω is an open set in the complex plane and $K \subset \Omega$ a compact subset such that $\Omega \backslash K$ has no components which are relatively compact in Ω, then every function f which is analytic (holomorphic) in a neighborhood of K can be approximated uniformly by analytic functions on Ω. This result should be considered as a typical example of holomorphic approximation. Further research in this area is due to Carleman (1927), who showed that for every continuous function $f\colon \mathbb{R} \to \mathbb{C}$ and for every error function $\varepsilon\colon \mathbb{R} \to (0, 1]$ one can find an analytic function g such that $|f - g| \leqslant \varepsilon$ on $\mathbb{R}$. Later, Mergelyan (1952) showed that a continuous function f on a compact set $K \subset \mathbb{C}$, which is analytic on the interior of K, can be approximated uniformly on K by entire functions (and thus also by polynomials) if and only if $\mathbb{C} \backslash K$ is connected. All these results deal with approximation on $\mathbb{R}$ or $\mathbb{C}$. If one replaces analytic functions by harmonic functions and if one looks for approximations on the n-dimensional Euclidean space $\mathbb{R}^n$, then one arrives at *harmonic approximation*, the theme of this book. The starting point of this kind of approximation is the following result by Walsh (1929): if $K \subset \mathbb{R}^n$ is a compact set such that $\mathbb{R}^n \backslash K$ is connected, then every function which is harmonic on an open set containing K can be approximated uniformly by a harmonic polynomial.

This booklet gives a systematic account of harmonic approximation. The theorems by Runge, Mergelyan, Carleman, and Walsh, which we mentioned earlier, are continuously used as an illustration, inspiration, and motivation for the results in harmonic approximation. At the end of each chapter the author gives appropriate credit and references for the results which were given in the chapter, and in addition some more recent developments are pointed out, many of which are by the author.

The first chapter deals with local harmonic approximation using functions which are harmonic on a neighborhood of the compact set on which one wants to approximate. The notion of *thin sets* is very important for this (comparable with the importance of connectedness for holomorphic approximation) and therefore this notion is briefly explained in a preliminary chapter. Then the harmonic analogues of the theorems of Runge and Mergelyan are given. In the second chapter the emphasis is on fusion of harmonic functions, which is a generalization of fusion of rational functions as described by Roth: for every pair of disjoint compact sets K_1 and K_2 in $\mathbb{C}^* = \mathbb{C} \cup \{\infty\}$, there exists a constant C such that for every pair of rational functions r_1, r_2 for which $|r_1 - r_2| < \varepsilon$ on a compact set $K \subset \mathbb{C}^*$ there exists a rational function r with $|r - r_1| < C\varepsilon$ on $K_1 \cup K$ and $|r - r_2| < C\varepsilon$ on $K_2 \cup K$. Chapter 3 starts with Arakelyan's generalization (1968) of Mergelyan's theorem. Then the corresponding result for harmonic approximation is given (Theorem 3.19). During this analysis a more general harmonic version of Runge's theorem from the first chapter is obtained. Carleman's generalization of

Weierstrass' theorem is the basis for Chapter 4. First the holomorphic version of Nersesyan (1971) is given, and then the related results for harmonic approximation follow. Chapter 5 deals with tangential approximation at infinity, where one tries to control the behavior of the error of approximation at infinity. This chapter complements the results from Chapter 3. Up to here the approximations are always in terms of harmonic functions. In Chapter 6 it is shown that many of the obtained results can be extended to superharmonic functions. Finally, the last two chapters contain some applications, such as a complete classification of all (unbounded) open sets Ω for which the Dirichlet problem can be solved (Chapter 7) and the existence of a nonconstant harmonic function for which the Radon transform vanishes and of other peculiar (pathological) harmonic functions (Chapter 8).

The author succeeded very well in writing a comprehensive book: the results are well motivated and illustrated (using the more commonly known results in holomorphic approximation), and at the end the author obtains some surprising applications. The reader is expected to have knowledge of complex analysis (with a background in subharmonic functions). The intended readership should consist of researchers in analysis, especially young researchers preparing a Ph.D. in analysis, but also postgraduates and more advanced researchers.

Walter Van Assche
E-mail: walter@wis.kuleuven.ac.be

P. Borwein and T. Erdélyi, *Polynomials and Polynomial Inequalities*, Graduate Texts in Mathematics **161**, Springer-Verlag, New York/Berlin, 1995, x + 480 pp.

Polynomials are surely among the simplest functions that one deals with. They are easy to differentiate or to integrate, the fundamental theorem of algebra tells us that any polynomial of degree n has precisely n complex zeros, and they have a very simple analytic description as a finite Taylor series. These are some of the reasons approximators like to use polynomials for approximation of functions. Polynomial approximation moreover works quite well for continuous functions on a compact set of the real line (Weierstrass' theorem). As the authors point out in their preface, virtually every branch of mathematics has its corpus of theory arising from the study of polynomials. Hence polynomials really deserve special attention and a book devoted especially to polynomials will be of interest to many people interested in mathematics.

This book by Borwein and Erdélyi gives an interesting approach to polynomials, with emphasis in the later chapters on polynomial inequalities. Only polynomials in one variable are treated, so those interested in polynomials in several variables will be disappointed. But then the authors are using the word polynomial in a very broad sense and very quickly put most of the exposition in terms of an extended notion of polynomials. To them, a Chebyshev system, a Markov system, or a Descartes system contains most of the essentials of polynomials. Müntz polynomials receive a lot of attention, as do rational functions with prescribed poles. The authors' treatment of Müntz polynomials is impressive and probably this subject is not treated in such detail elsewhere.

The presentation of the book is unusual. Often the text gives an introduction to a particular aspect of the theory and then gives much more in the form of exercises. As an example, the Stone–Weierstrass theorem is given as exercise E.2 on p. 161, and the interested reader is led through the proof in seven steps by means of useful hints. This approach of presenting relevant results in such a way that the reader needs to work out the proof by himself is used throughout the book. The book is very similar in spirit to one of my favorite classics, *Problems and Theorems in Analysis*, by Pólya and Szegő. This presentation is very appealing